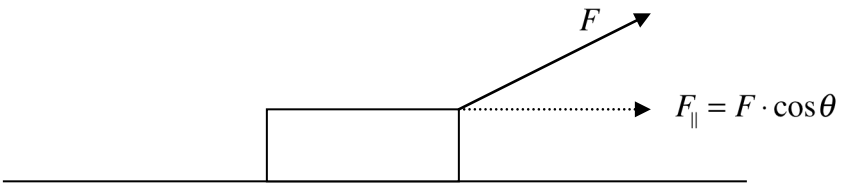
Work Done by a Constant Force

Work done on an object by a constant force is defined to be the product of the magnitude of the displacement times the component of the force parallel to the displacement.

$$W = F_{\parallel} \cdot d$$



So we can write:

$$\boxed{W = F \cdot d \cdot \cos\theta}$$

Units: Joules $(1\ J = 1\ N \cdot m)$

A force can be exerted on an object and yet do no work. For example, if you hold a heavy bag of groceries in your hands at rest, you do no work on it. You do exert a force on the bag, but the displacement of the bag is zero, so the work done by you on the bag is $W = 0$. You need both a force and a displacement to do work.

You also do no work on the bag of groceries if you carry it as you walk horizontally across the floor at constant velocity. No horizontal force is necessary to move the bag at a constant velocity. You do exert an upward force on the bag equal to its weight, but this upward force is perpendicular to the displacement and thus has nothing to do with that motion.

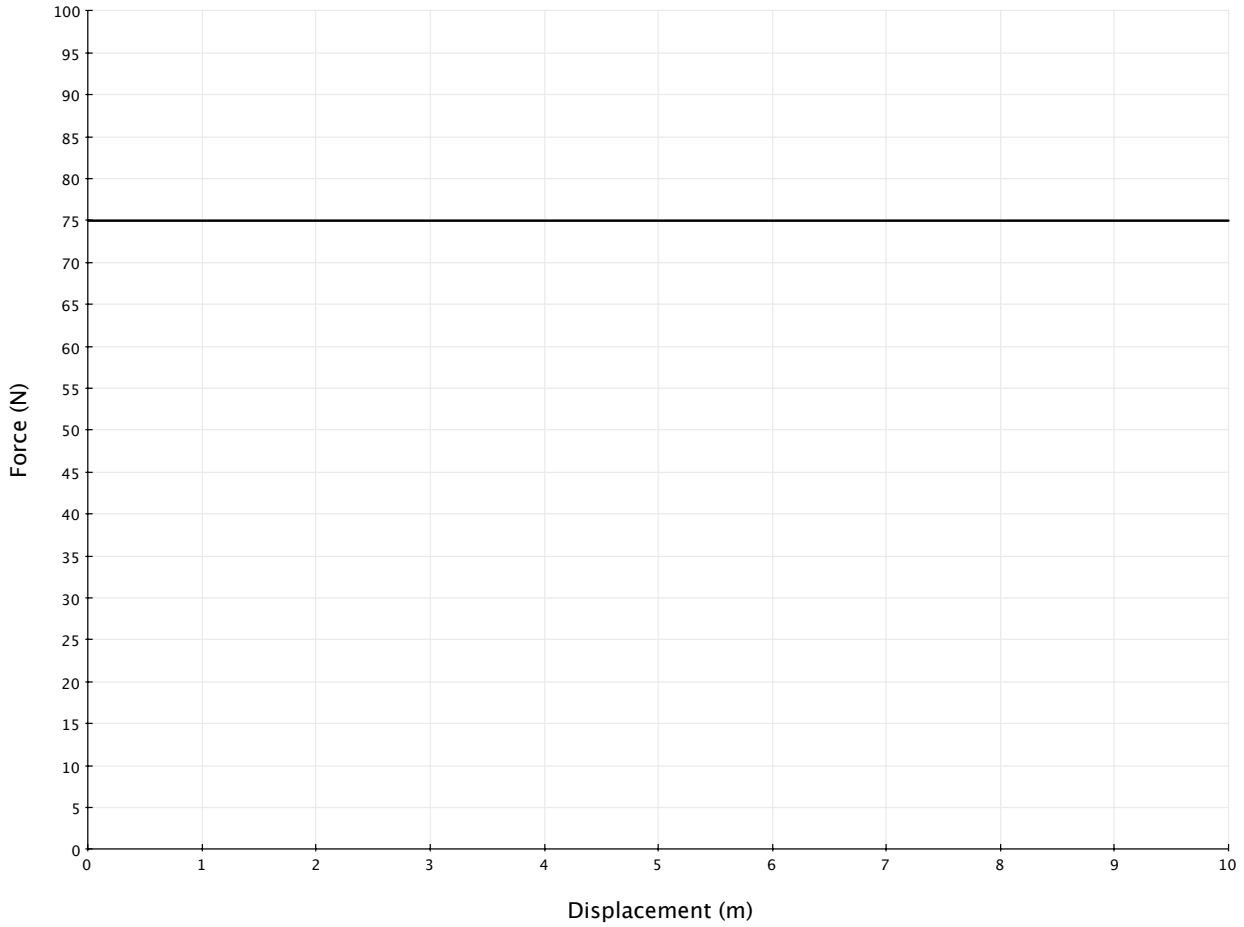
Thus, when a particular force is perpendicular to the displacement, no work is done by that force.

Example 1 Work done on a crate.
A person pulls a $50.0\ kg$ crate $40.0\ m$ along a horizontal floor by a constant force $F_p = 125\ N$, which acts at an angle of $37°$ above the horizontal. The floor is rough and exerts a frictional force, $F_f = 50.0\ N$. Determine (a) the work done by each force acting on the crate, and (b) the net work done on the crate.

Example 2 Work on a backpack.
Determine (a) the work a hiker must do on a $15.0\ kg$ backpack to carry it up a hill of height $h = 10.0\ m$, (b) the work done by gravity on the backpack, and (c) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity.

Work Done by a Varying force

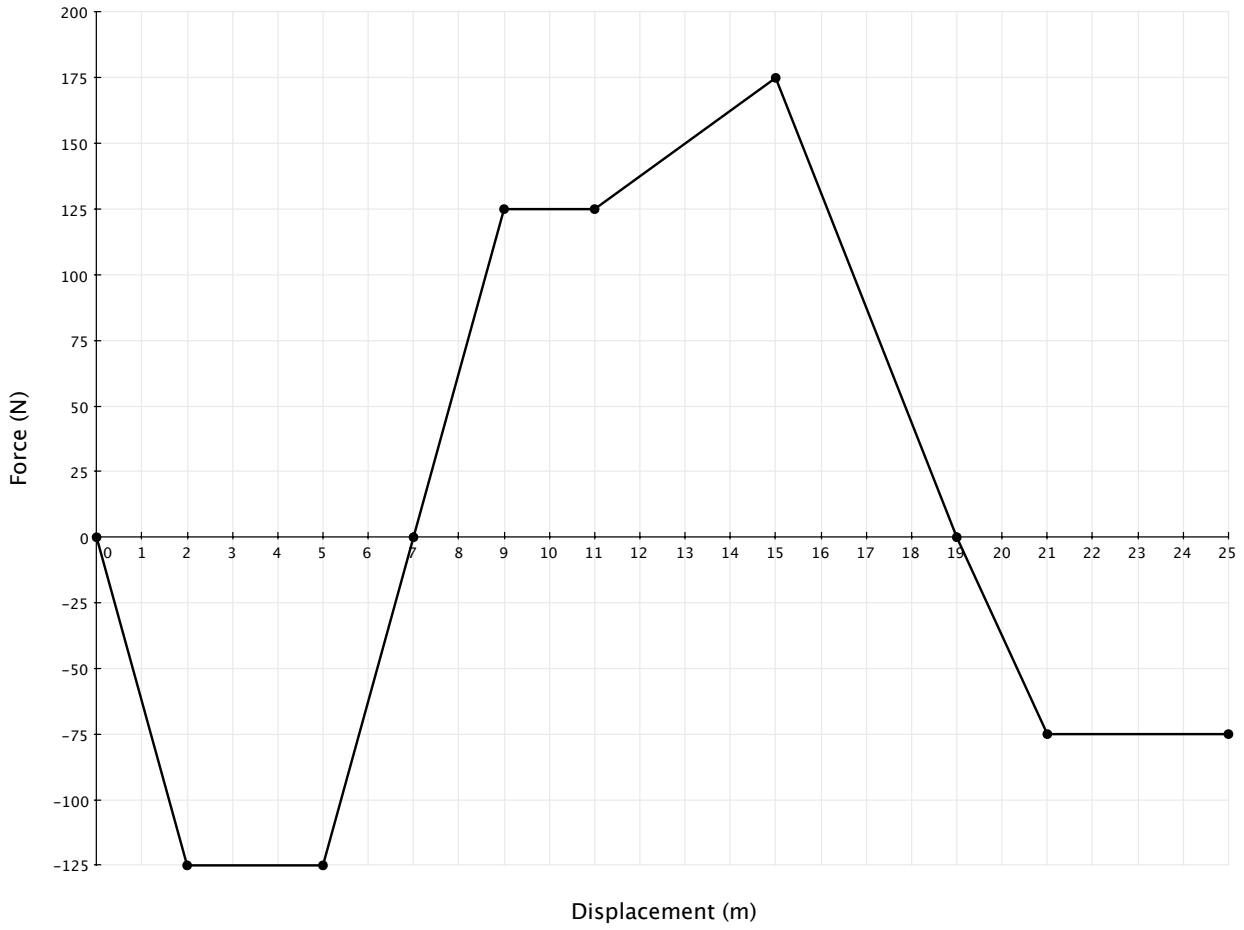
If the force acting on an object varies, the work must be calculated from the area under a force vs. displacement graph.



Area that is above the *x*-axis is considered positive work. Area that is below the *x*-axis is negative work.

Example 3

The graph below shows how the force acting on an object varies with displacement. Determine the total work done on the object by this force.



Homework

Energy Worksheet #1

Energy Worksheet #1

1. A force of $800\ N$ is needed to push a car across a lot. Two students push the car $40\ m$. How much work is done? ($32000\ J$)

2. How much work is done lifting a $60\ kg$ crate a vertical distance of $10\ m$? ($5880\ J$)

3. A person carries a $34\ N$ package from the ground floor to the fifth floor of an office building, or $15\ m$ upward. How much work does the person do, against the force of gravity, to move the package? ($510\ J$)

4. What work is done to lift a $49\ kg$ crate a distance of $10\ m$? ($4800\ J$)

5. A worker carries cement blocks, weighing $150\ N$ each, up a ladder onto a scaffold $8.0\ m$ high. The worker carries 2 blocks per minute. How much work is done by the worker in
   a. 10 minutes? ($24000\ J$)
   b. 1 hour? ($144000\ J$)

6. The hammer of a pile driver has a mass of $100\ kg$. The machine's engine lifts it to a height of $5.0\ m$ every 10 seconds.
   a. How much work must the machine do to lift the hammer? ($4900\ J$)
   b. How much work does the machine do in 1 minute?($29000\ J$)

7. A force of $600\ N$ is applied to a metal box to pull it $15\ m$ across a floor. The rope used to pull the box is held at an angle of $46°$ with the floor. How much work is done? ($6250\ J$)

8. A person uses a rope to pull a $1000\ kg$ boat $50\ m$ along a wharf. The rope makes an angle of $45°$ with the horizontal. If a force of $40\ N$ is used to move the boat, how much work is done? ($1410\ J$)

9. It takes $12000\ J$ of work to pull a loaded sled weighing $800\ N$ a distance of $200\ m$. To do this, a force of $120\ N$ is exerted on a rope, which makes an angle with the horizontal. At what angle is the rope held? ($60°$)

10. A cable attached to a small tractor pulls a barge through a canal lock. The cable makes an angle of $30°$ with the direction in which the barge is moving and has a tension of $2500\ N$.
    a. What force moves the barge along the lock? ($2170\ N$)
    b. If the lock is $200\ m$ long, how much work is done to get the barge through the lock? ($433000\ J$)

11. Due to friction, a force of $400\ N$ is needed to drag a wooden crate across a floor. The rope tied to the crate is held at an angle of $56°$ with the horizontal.
    a. How much tension is needed in the rope to move the crate? ($715\ N$)
    b. What work is done if the crate is dragged $25\ m$? ($10000\ J$)

12. A student librarian picks up a $0.95\ N$ book from the floor to a height of $1.25\ m$. She carries the book to the shelves and places it on a shelf that is $2.0\ m$ high. How much work has been done on the book, against the force of gravity? ($1.9\ J$)

13. What is the amount of work required to push a $1000\ kg$ car $245\ m$ up a $22.5°$ incline at a constant speed?
    a. Ignore friction. ($920000\ J$)
    b. Assume the coefficient of friction is 0.30. ($1600000\ J$)

14. Eight bricks, each $6.0\ cm$ thick with a mass of $1.2\ kg$, lie flat on a table. How much work is required to stack them one on top of another? ($20\ J$)

15. The force on a particle varies as shown below. Determine the work done by this force to move the particle from
    a. $x = 0$ to $x = 10$ ($2800\ J$)
    b. $x = 0$ to $x = 15$ ($2050\ J$)