Power and Energy

As current passes through a lamp, charges are moving from a higher potential to a lower one. In other words, the charges are losing energy as it is converted into heat and light in the filament of the lamp. The amount of energy released by a charge q as it moves through a potential difference V across the lamp is

$$\Delta PE_E = qV$$

The rate at which energy is released, or the **power** P, is

$$P = \frac{\Delta PE_E}{t}$$

Combining the two equations above, we get

$$P = \frac{(qV)}{t} = V\frac{q}{t}$$

But $\frac{q}{t}$ is just the current I, so we have

$$P = IV$$

When the potential difference is measured in volts and the current in amperes, their product gives the power in **watts**.

In a resistor, the energy dissipated appears as thermal energy. This effect is used in appliances such as electric stoves, hair dryers, and heaters. In an incandescent lamp, the energy delivered to the filament raises its temperature so high that light is emitted. In other circuits, the energy may take on different forms. For example, the energy may appear as mechanical work done by a motor, as sound from a loudspeaker, or as stored chemical energy in a battery when it is being recharged.

Note: Conversion from electrical to mechanical energy is never 100% efficient. The difference appears as heat.

Example 1
A typical electric hair dryer designed to operate on a $120\ V$ household circuit is rated at $1500\ W$. What is the resistance of the dryer?

Example 2
What is the power consumption of an electric iron if its resistance is $13.1\ \Omega$ and it operates on a household circuit? Assume an effective household voltage of $120\ V$.

In both of these examples, we used two equations sequentially to obtain an answer. We can simplify this process greatly by combining these equations. For example, if we use Ohm's law to eliminate current in the equation for power:

$$P = IV \text{ and } I = \frac{V}{R}$$

Therefore,

$$P = \frac{V}{R} \cdot V$$

$$\boxed{P = \frac{V^2}{R}}$$

Similarly, we can use Ohm's law to eliminate voltage in the equation for power:

$$P = IV \text{ and } V = IR$$

Therefore,

$$\begin{aligned} P &= IV \\ &= I(IR) \end{aligned}$$

$$\boxed{P = I^2 R}$$

These two equations describe the rate of transfer of electrical energy to heat energy in a resistor. Both of these equations are known as **Joule's law**.

Example 3
A piece of wire has a resistance of $30\ \Omega$. How much power is dissipated in the wire if it carries a current of $0.50\ A$?

Example 4
In the stairwell of a ten-story building, there are two continuously burning $75\ W$ safety lamps for each floor.

a) What is the total energy (in kilowatt-hours) used in one year?

b) What will it cost to use the lamps for a year if the cost of electricity is $\$0.078\ /\ kWh$?

Note: The SI unit of energy is the joule or watt-second $(1\ J = 1\ W \cdot s)$. However, this unit is inconveniently small because in our homes we consume energy at the rate of kilowatts and do so for hours or days. The kilowatt-hour (kWh) is a common unit for energy. It is defined as the energy consumed in one hour $(3.6 \times 10^3\ s)$ by operating at a constant power of $1\ kW$.

$$1\ kWh = 3.6 \times 10^6\ J$$

Circuits Worksheet #3

1. A $150\ W$ lamp operates at a rated voltage of $120\ V$.

    a) How much current is drawn by the lamp? ($1.25\ A$)

    b) What is the resistance of the lamp when operating? ($96\ \Omega$)

2. A flashlight lamp connected to a battery that provides $1.4\ V$ draws a current of $0.10\ A$. What electric power is used by the lamp? ($0.14\ W$)

3. The label on a toaster reads $800\ W$ at $120\ V$. How much current does it draw? ($6.67\ A$)

4. A drilling machine operates with an electric power consumption of $840\ W$. How much does it cost to operate the machine continuously for eight hours if the electricity costs $\$0.080 / kWh$? ($0.54)

5. A refrigerator is equipped with a motor that draws $100\ W$ but operates only 25% of the time. What is the cost of operating the refrigerator for 30 days if electricity costs $\$0.080 / kWh$? ($1.44)

6. A $150\ W$ street lamp is operated for 12 hours a day. How much energy does it take to operate the lamp for 30 days? Express your answer in kilowatt-hours and in joules. ($54\ kWh$, $1.9 \times 10^8\ J$)

7. How much does it cost to operate a $100\ W$ lamp 8 hours a day for 30 days if electricity costs $\$0.075 / kWh$? ($1.80)

8. A $100\ \Omega$ resistor is rated at $1.00\ W$ maximum power capacity.

    a) What is the maximum voltage that can be applied across the resistor without exceeding its maximum power rating? ($10\ V$)

    b) What is the current at this voltage? ($0.1\ A$)

9. A $40\ W$ lamp operates at a rated voltage of $130\ V$.

    a) What is the resistance of the lamp? ($422.5\ \Omega$)

    b) How does that compare to the "cold" resistance of $31\ \Omega$ measured at very low voltage?

    c) What does this calculation tell you about the relationship between resistance and temperature for the material of the filament?